Partially Filled Viscous Ring Nutation Damper

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The problem of a spinning satellite with a partially filled viscous ring nutation damper is investigated. It is shown that there are two distinct modes of motion, previously named the nutation synchronous mode and the spin synchronous mode. An approximate solution for the nutation angle time history is obtained and time constants for the two modes are given as a function of suitable dimensionless parameters. Comparison is then made with the exact solution obtained by numerical integration of the exact equations of motion. From a consideration of the fluid dynamics several methods are developed for determining the damping constant.

1. Introduction

THE first analysis of a partially filled viscous ring damper on a spinning satellite was performed by Carrier and Miles.[1,2] They assumed that the motion of the damper did not appreciably affect the precession rate of the satellite but acted only as a source of energy dissipation. With this assumption the motion of the fluid on the tube was then treated as a fluid mechanics problem and an approximate solution to the Navier-Stokes equations was obtained. In this mode of motion the fluid is spread out over the outer part of the tube. They then showed that as the nutation angle increases gaps will begin to appear in the fluid and finally somewhere between $\frac{1}{2}°$ and 1° the fluid will begin to behave as a rigid slug. The angle of transition between the modes is dependent on the parameters of the system. At the nutation angles where the fluid behaves as a rigid slug the problem was treated as boundary-layer flow over a flat plate with the width of the plate being equal to the perimeter of the tube. However the analysis did not completely treat the problem because, as will be shown later, there are two distinctive modes of motion when the fluid behaves as a slug and in one of these modes the boundary-layer flow assumption is not valid. On some satellites, such as Helios, which employ this type of nutation damper, the center of the ring is offset from the spin axis so that the fluid will always behave as a rigid slug. The next analysis, which was performed by Cartwright et al.,[3,4] modelled the fluid as a particle of equal mass moving in a tube resisted by a viscous damping force. Their analysis revealed that there are two distinct modes of motion which they named the nutation synchronous mode and the spin synchronous mode. Although there were some minor errors in their equations of motion they correctly analyzed the nutation synchronous mode but failed to analyze the spin-synchronous mode. After these analyses several dampers using mercury were used on satellites.

Interest was revived in this problem when the failure of the ATS-5 satellite was traced to energy dissipation which resulted from fluid motion in the heat pipes. The heat pipes were behaving as partially filled viscous ring dampers. A ring damper using mercury is planned for the Helios satellite and during a portion of the flight the satellite will be spinning about its axis of minimum moment of inertia. These factors make it necessary to be able to predict more accurately the energy dissipation resulting from this type of damper.

Alfriend[5] approached the problem in the same manner as Cartwright[3,4] and obtained approximate equations describing the motion in both modes of motion. The agreement between the approximate equations and exact equations was excellent. When the problem is approached in this manner it must be accompanied with a method for determining the damping constant. Recently Leibold[6] analyzed the problem using the same approach as Alfriend[5] and Cartwright[3,4] and suggested assuming steady flow in a straight pipe as a means of calculating the damping constant.

The approach taken in this analysis is to assume that the fluid is behaving as a rigid slug. Since the fluid may fill up to 50% of the tube the slug is not modeled as a point mass as in previous studies but is modeled as a rigid slug of finite length. The equations of motion are developed in terms of dimensionless variables and approximate time constants are obtained in terms of a suitable set of dimensionless parameters. These approximate solutions are then compared with those obtained from numerical integration of the exact equations of motion. The time constant obtained by Miles for very small nutation angles is then given. The problem of determining the damping constant by considering the fluid dynamics is then attacked and comparisons of several methods are given with the drawbacks of the methods pointed out.

2. Problem Solution

The equations of motion of a rigid slug moving in a tube which encircles the spin axis of a symmetric satellite are developed in

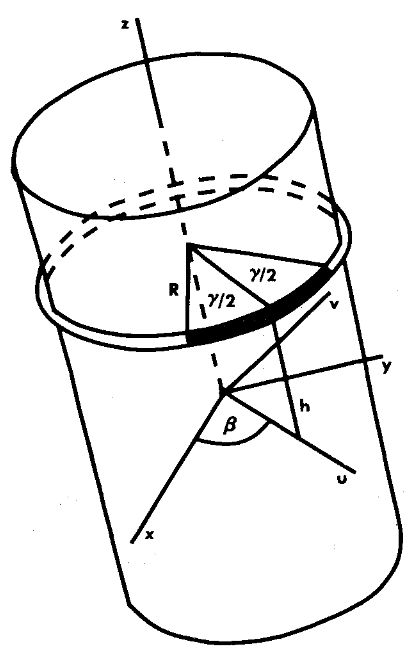

Fig. 1 Mathematical model and coordinate systems.

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Appendix A. p, q, and r are the dimensionless components of the angular velocity along the u, v, and z axes. Referring to Fig. 1 the u, v, and z axes rotate relative to the body such that the u axis passes through the center of mass of the slug. β is the angular position of the slug measured from the x-axis, i.e., it is the angle between the x and u axes. The dimensionless parameters of the system are ε, b, σ, η, and γ where ε is a small parameter which is the ratio of the moment of inertia of the tube filled with fluid to the transverse moment of inertia of the satellite, b is the ratio of the height of the ring above the satellite center of mass to the radius of the ring, σ is the ratio of the spin and transverse moments of inertia, η is a dimensionless damping constant, and γ is the angle of fill of the fluid in the ring. A dimensionless time $\tau = \Omega t$ is used where Ω is the spin rate of the satellite.

Before developing the solution for the motion of the slug the equation for the nutation angle will be developed. The nutation angle θ is given by

$$\tan\theta = H_t/H_z \tag{1}$$

where H_t is the transverse component of the angular momentum, $H_t^2 = H_u^2 + H_v^2$, and H_z is the spin axis component of the angular momentum. Differentiation of Eq. (1) gives

$$\theta' = H_t'/H_z = -H_z'/H_t = (pH_v - qH_u)/H_t \tag{2}$$

Substitution of Eqs. (A1) for H_u and H_v gives

$$\theta' = [(\sin\gamma/\gamma)p + bk(r+\beta')]\varepsilon\bar{\gamma}q/(p^2+q^2)^{1/2} + O(\varepsilon^2) \tag{3}$$

where $k = [\sin(\gamma/2)]/(\gamma/2)$.

The advantage of using Eq. (3) to determine θ rather than Eq. (1) should now be obvious. Using Eq. (3) to determine θ to $O(\varepsilon)$ only requires the first approximation of p, q, r, and β, whereas if Eq. (1) was used p, q, r, and β would have to be calculated through the second approximation, i.e., to $O(\varepsilon)$, in order to obtain θ to $O(\varepsilon)$.

Neglecting terms of $O(\varepsilon)$ the equations of motion, Eq. (A9) become

$$r = 1$$

$$\begin{aligned} p' + (\lambda - \beta')q &= 0 \\ q' - (\lambda - \beta')p &= 0 \end{aligned} \tag{4}$$

$$\beta'' + \eta\beta'' + (\sin\gamma/\gamma)pq + b\sigma kq = 0 \tag{5}$$

where $\lambda = \sigma - 1$ is the nutation frequency. The solution for p and q is

$$\begin{aligned} p &= -\omega_t \cos(\lambda\tau - \beta) \\ q &= -\omega_t \sin(\lambda\tau - \beta) \end{aligned} \tag{6}$$

The equation for θ becomes

$$\theta' = [(\sin\gamma/\gamma)\omega_t \cos(\lambda\tau-\beta) - bk(1+\beta')]\varepsilon\bar{\gamma}\sin(\lambda\tau-\beta) + O(\varepsilon^2) \tag{7}$$

Since

$$\tan\theta = H_t/H_z = \omega_t/\sigma + O(\varepsilon) \tag{8}$$

we can use Eq. (8) to substitute for ω_t. We now need to obtain the solution for β.

Damper Motion

A symmetric rigid body which is spinning about its axis of symmetry has a constant nutation angle when no damping is present. The transverse angular velocity vector ω_t rotates at a rate of $\sigma\Omega\cos\theta$ and the body rotates relative to ω_t at a rate of $(1-\sigma)\Omega$. When no damping is present the center of mass of the fluid slug will be flung outward as far as possible which will be along ω_t or the plane formed by **H** and the z axis, hereafter called the nutation plane. The fluid slug will then be moving at a constant rate of $(1-\sigma)\Omega$ with respect to the body. Introduction of a small amount of damping causes the center of mass of the fluid slug to move off the nutation plane to an equilibrium position where a component of the centrifugal force balances the friction force. This type of motion is called "nutation-synchronous" motion. In this mode the fluid slug is moving at a constant rate with respect to the body, hence the energy dissipation rate is a constant. If $\sigma > 1$ the nutation angle decreases which causes a decrease in the centrifugal force and the fluid slug center of mass moves further from the nutation plane. Eventually the centrifugal force is not large enough to balance the damping force and the fluid slug begins to be dragged around with the body while oscillating in the tube. This type of motion is called "spin-synchronous" motion. The terms nutation-synchronous and spin-synchronous were first introduced by Cartwright et al.[3,4] A typical nutation angle time history showing the two modes of motion is shown in Fig. 2. Although the angle of transition between the two modes in Fig. 2 is less than the angle for which the fluid begins to behave as a slug this transition angle can easily be 10°–15° for a different set of parameters. However it is entirely possible that the spin-synchronous mode may not exist if the ring is not offset from the spin axis.

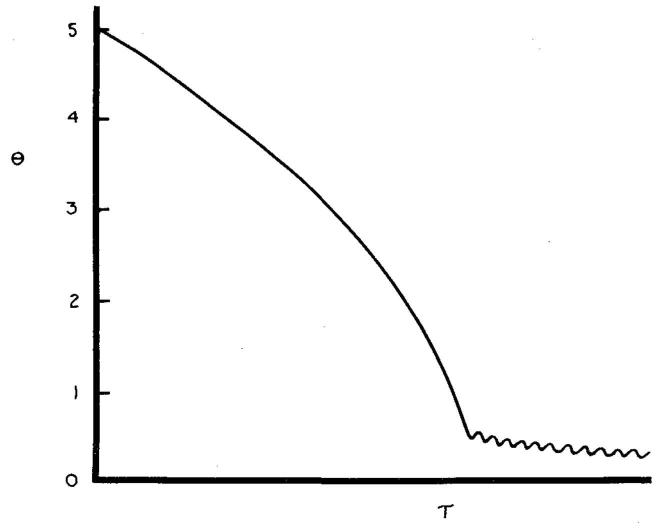

Fig. 2 Typical nutation angle decay.

The purpose now is to determine β and consequently θ in these two modes of motion as a function of the dimensionless parameters ε, η, σ, b, and γ.

Nutation-Synchronous Mode

Let α measure the position of the center of mass of the fluid slug with respect to the nutation plane. Assuming that at $\tau = 0$, $\beta = 0$ then

$$\alpha = \beta - \lambda\tau \tag{9}$$

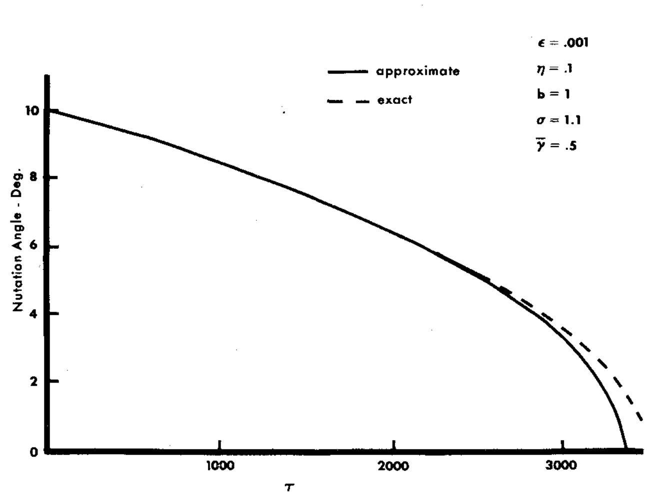

Fig. 3 Comparison of exact and approximate nutation angle time histories for the nutation synchronous mode.

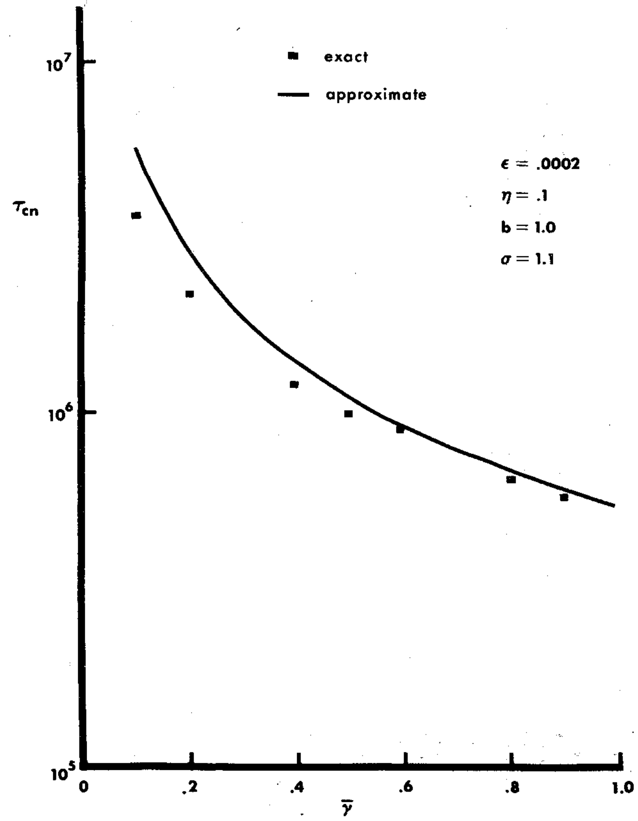

Fig. 4 τ_{cn} vs $\bar{\gamma}$.

Equation (5) becomes

$$\alpha'' + \eta\alpha' + b\sigma k\omega_t \sin\alpha\left[1 - \frac{\omega_t \cos(\gamma/2)\cos\alpha}{b\sigma}\right] = -\eta\lambda \quad (10)$$

A particular solution of this equation is $\alpha = \alpha_e$ where

$$\sin\alpha_e\left[1 - \frac{\tan\theta\cos(\gamma/2)\cos\alpha_e}{b}\right] = -\frac{\eta(\sigma-1)}{b\sigma^2 k} \quad (11)$$

where $\omega_t = \sigma\tan\theta$ has been used. Thus the fluid slug will remain in the nutation-synchronous mode as long as Eq. (11) is satisfied. Once θ becomes small enough so that $\alpha_e = \pm\pi/2$ the fluid slug goes into the spin-synchronous mode. The transition angle θ_t from one mode to the other is

$$\tan\theta_t = \eta|\sigma - 1|/(b\sigma^2 k) \quad (12)$$

Substitution of the solution for α into the nutation angle rate equation, Eq. (7), and neglecting terms of $O(\varepsilon^2)$ gives

$$\tan\theta\,\theta' = -\varepsilon\eta\bar{\gamma}(\sigma-1)/\sigma \quad (13)$$

which has the solution

$$\cos\theta = \cos\theta_0 \exp(\tau/\tau_{cn}) \quad (14)$$

where

$$\tau_{cn} = \sigma/\varepsilon\bar{\gamma}\eta(\sigma-1) \quad (15)$$

Thus in the nutation synchronous mode the cosine of the nutation angle, not the nutation angle, exhibits exponential behavior. No small angle approximation has been made, hence Eq. (14) is valid for $0 < \theta < \pi/2$. For small θ the nutation angle time history can be approximated by

$$\theta = (\theta_0{}^2 - 2\tau/\tau_{cn})^{1/2} \quad (16)$$

A comparison of θ given by Eq. (14) and θ obtained from numerical integration of the equations of motion, Eqs. (A9), is given in Fig. 3. A comparison of τ_{cn} given by Eq. (15) and an "exact" time constant is given in Figs. 4 and 5. The "exact" time constant is obtained by assuming exponential behavior of $\cos\theta$ for the solution obtained by numerical integration of Eqs. (A9) and calculating a time constant. Figures 3–5 show that the approximate solutions given by Eqs. (14) and (15) are good.

If there are N dampers the time constant is

$$\tau_{cn} = \sigma\Big/\left[(\sigma-1)\sum_{i=1}^{N}\varepsilon_i\bar{\gamma}_i\eta_i\right] \quad (17)$$

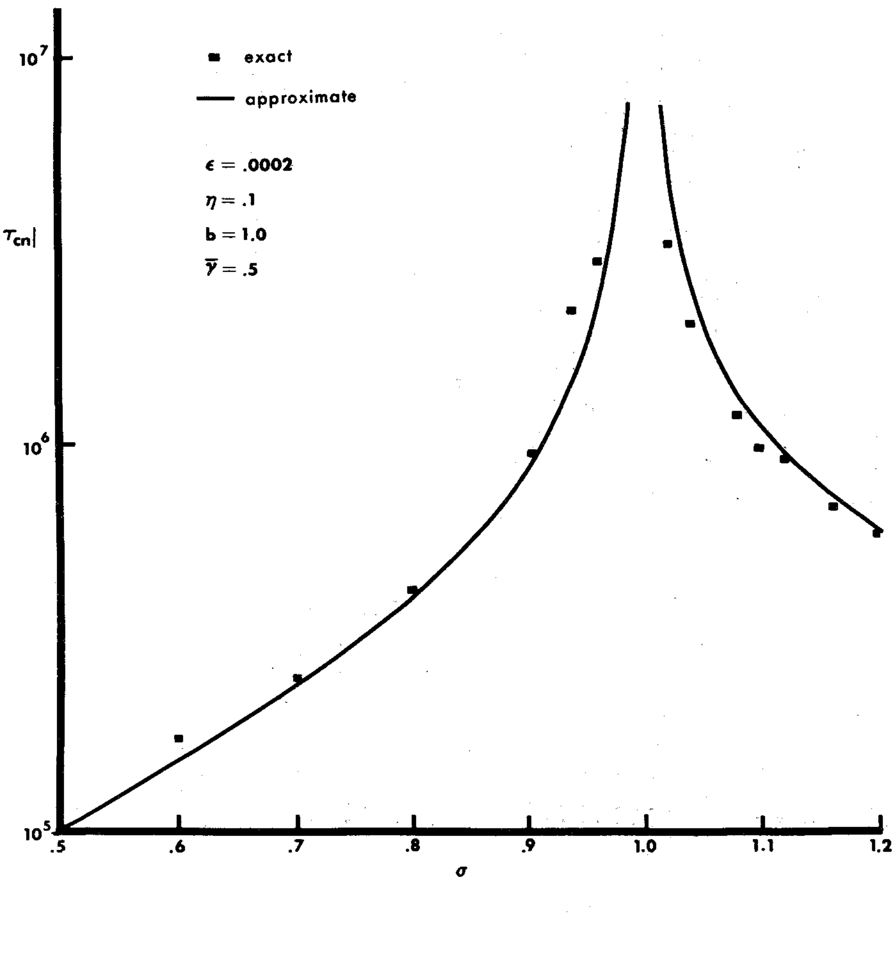

Fig. 5 τ_{cn} vs σ.

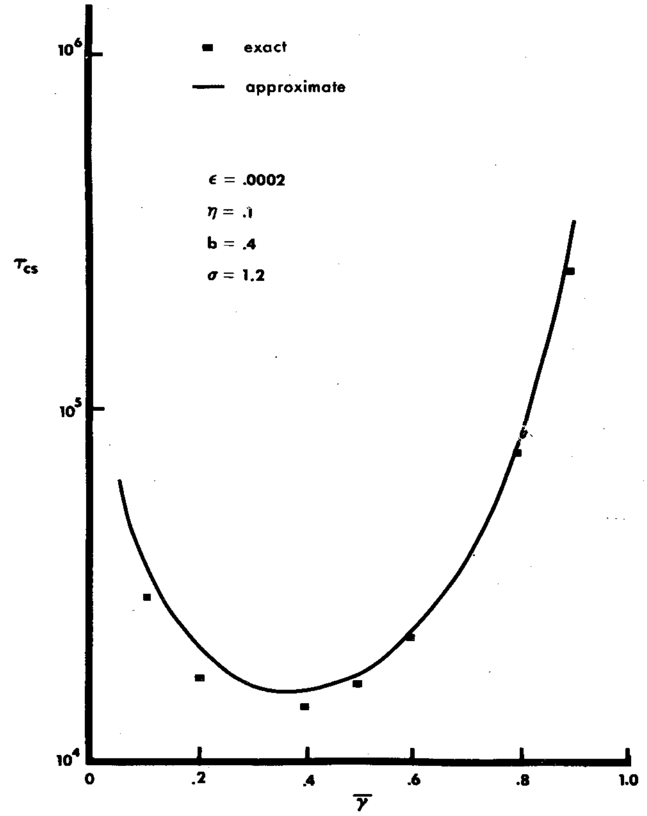

Fig. 6 τ_{cs} vs $\bar{\gamma}$.

Spin-Synchronous Mode

Substitution of the solution for p and q into Eq. (5) gives

$$\beta''+\eta\beta'+b\sigma k\omega_t \sin(\beta-\lambda\tau)\left[1-\frac{\omega_t\cos(\gamma/2)\cos(\beta-\lambda\tau)}{\sigma b}\right]=0 \quad (18)$$

Since the spin synchronous mode occurs for the smaller nutation angles $\omega_t \ll 1$, thus the last term within the brackets in Eq. (18) will be dropped. An approximate steady-state solution of Eq. (18), obtained by taking the first iterate of a Picard iteration, is

$$\beta-\beta_o=(bk\sigma^2/\lambda)\tan\theta[E\sin(\beta_o-\lambda\tau)+F\sin(\beta_o-\lambda\tau)] \quad (19)$$

where β_o is the initial value of β and

$$E=\lambda/(\lambda^2+\eta^2)$$
$$F=-\eta/(\lambda^2+\eta^2) \quad (20)$$

A second iterate in the Picard iteration scheme would show the slow movement of the slug around the tube, but this second iterate is not needed to determine θ. The first iterate is obtained by assuming $\beta=\beta_o$ in the $\sin(\beta-\lambda\tau)$ and $\cos(\beta-\lambda\tau)$ terms. The basis for this assumption is that the change in β is small compared to $\lambda\tau$.

The nutation angle differential equation is

$$\theta'=[-(\sin\gamma/\gamma)\omega_t\cos(\beta-\lambda\tau)+(1+\beta')bk]\varepsilon\bar{\gamma}\sin(\beta-\lambda\tau) \quad (21)$$

Assuming that θ is small enough so that terms of $O(\theta^2)$ can be neglected and assuming that the change in β is small so that $\sin(\beta-\beta_o)=(\beta-\beta_o)$ and $\cos(\beta-\beta_o)=1$. Equation (21) reduces to

$$\theta'+\theta[(1/\tau_{cs})+\kappa_1\cos 2\lambda\tau+\kappa_2\sin 2\lambda\tau]=-(\varepsilon b/\pi)\sin(\gamma/2)\sin\lambda\tau \quad (22)$$

where κ_1 and κ_2 are constants and

$$\tau_{cs}=\frac{2(\sigma-1)[(\sigma-1)^2+\eta^2]}{\varepsilon b^2\sigma^3\eta\bar{\gamma}[\sin(\gamma/2)/(\gamma/2)]^2} \quad (23)$$

The solution of Eq. (22) is an infinite series but the κ_1 and κ_2 terms contribute nothing to the exponential decay of the solution. The important part of the solution is

$$\theta=\left(\theta_o+\frac{\lambda}{\lambda^2+1/\tau_{cs}^2}\right)\exp(-\tau/\tau_{cs})+\left(\frac{1}{\tau_{cs}}\sin\lambda\tau-\lambda\cos\lambda\tau\right)\Big/(\lambda^2+1/\tau_{cs}^2) \quad (24)$$

Thus the solution is the superposition of an oscillation and an exponential decay with a time constant given by Eq. (23). A comparison of the time constant given by Eq. (23) and an "exact" time constant is given in Figs. 6 and 7. The "exact" time constant was obtained by numerically integrating the exact equations of motion, assuming exponential behavior for maximum values of θ during each oscillation and calculating the time constant. Again the approximate solution developed is a good one.

When there are N dampers the time constant becomes

$$\frac{1}{\tau_{cs}}=\frac{2(\sigma-1)}{\sigma^3}\sum_{i=1}^{N}\left\{\frac{[(\sigma-1)^2+\eta_i^2]}{\varepsilon_i b_i^2\eta_i\bar{\gamma}_i[\sin(\gamma_i/2)/(\gamma_i/2)]^2}\right\} \quad (25)$$

3. Calculation of the Damping Constant η

In the preceding section approximate equations were developed for the nutation angle behavior and corresponding time constants. Comparison with the results from numerical integration of the exact equations of motion has shown that the approximate equations developed are good approximations of the time constants. For a given system all of the required dimensionless parameters are well defined except for the damping constant η or c_d. η is related to the kinematic viscosity, thus a consideration of the fluid dynamics of the problem is necessary. Since the motion of the fluid slug is different in the two modes the fluid dynamics problem must be considered for each mode.

Nutation-Synchronous Mode

In the nutation synchronous mode the velocity of the fluid slug is constant with respect to the satellite or tube. As a result two approaches have been suggested for calculating η. The approach suggested by Leibold[6] is to model the motion of the fluid as

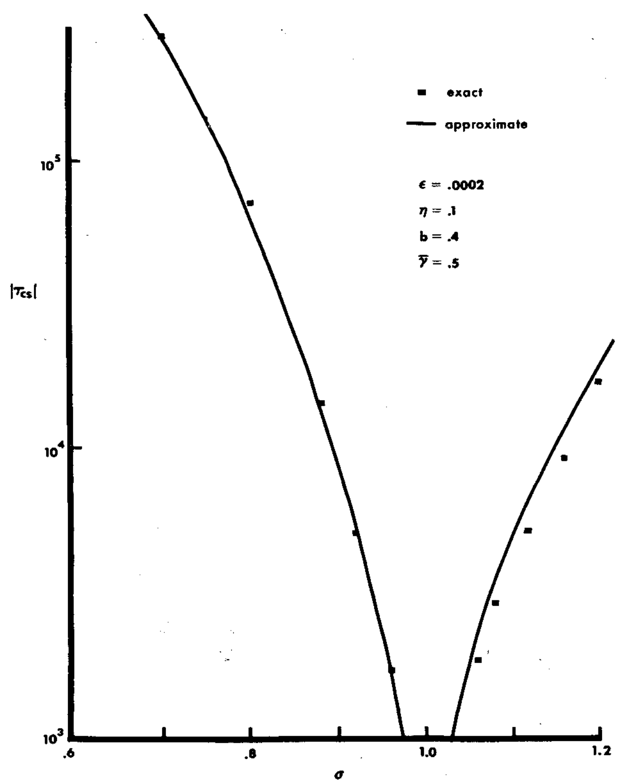

Fig. 7 τ_{cs} vs σ.

steady flow in a pipe and the second approach used by Carrier and Miles[1,2] is to model it as boundary-layer flow over a flat plate with the width of the plate being the perimeter of the pipe. Both approaches must be considered for both laminar and turbulent flow.

A) Steady Flow in a Pipe

1) *Laminar flow:* The development of steady flow in a straight pipe can be found in almost any standard fluid mechanics text such as Ref. 7.

Solution of the Navier-Stokes equations with a flux of

$$Q = \pi a^2 R\dot{\beta} \tag{26}$$

where $R\dot{\beta}$ is the velocity of the fluid slug relative to the ring and a is the radius of the pipe, gives

$$u = 2R\dot{\beta}[1-(r/a)^2] \tag{27}$$

The shear stress at any point is

$$\tau = \mu \, \partial u/\partial r \tag{28}$$

where μ is the viscosity. Thus the total viscous force is

$$F = (2\pi a)(R\gamma)\tau|_{r=a} = 8\pi\mu R^2\dot{\beta}\gamma \tag{29}$$

Equating this to the force determined from the dynamic analysis, i.e.,

$$F = c_d v = c_d R\dot{\beta} = \eta m\Omega R\dot{\beta} \tag{30}$$

gives

$$\eta = 8(v/a^2\Omega) \tag{31}$$

where v is the kinematic viscosity. The quantity $(a^2\Omega/v)$ is a Reynolds number but is not the standard Reynolds number for steady flow in a pipe. The Reynolds number is

$$Re = 2aR\dot{\beta}/v \tag{32}$$

Since the flow is laminar for $Re < 2000$, Eq. (31) holds for $Re < 2000$.

The preceding analysis assumed flow in a straight pipe but we have flow in a curved pipe. The correction factor for flow in a curved pipe as given by Schlichting[8] is

$$\tau/\tau_o = 0.1064[Re(a/R)^{1/2}]^{1/2} \tag{33}$$

where τ_o is the shear stress in the straight pipe and τ is the shear stress in the curved pipe. Equation (32) is valid for $10^{1.6} < (a/R)^{1/2} Re < 10^3$. For $Re = 2000$ and $(a/R) = \frac{1}{100}$ the increase in shear stress is 50%. Therefore one should take into account the curvature of the pipe. The damping constant for laminar flow then becomes

$$\eta = 1.2(v/a^2\Omega)^{1/2}(R/a)^{1/4}|\sigma-1|^{1/2} \tag{34}$$

The assumption here is that we have steady flow in a pipe, but a certain length is required for steady flow to develop. For flow from a cistern into a pipe this length (Ref. 7, p. 301) is

$$\delta = 0.0575a\, Re \tag{35}$$

which for $Re = 1500$ is $\delta = 86a$. But the length of the fluid in many cases may not be much more than $86a$. Thus the length of fluid required for steady flow to develop may be about equal to the length of the fluid. This is the error in assuming steady flow.

2) *Turbulent flow:* Blasius (Ref. 7, p. 339) developed for steady turbulent flow in a straight pipe the following empirical result for the shear stress

$$\tau = 0.0791\, Re^{-1/4}(\tfrac{1}{2}\rho u_m^2), \quad 2\times10^3 < Re < 10^5 \tag{36}$$

where u_m is the mean velocity $R\dot{\beta}$. Using the same procedure as before to calculate the damping constant one obtains

$$\eta = 0.133(v/a^2\Omega)^{1/4}(R/a)^{3/4}|\sigma-1|^{3/4} \tag{37}$$

The correction factor to take into account the fact that the flow is in a curved pipe is

$$\tau/\tau_o = 1+0.075\, Re^{1/4}(a/R)^{1/2} \tag{38}$$

This correction factor is smaller than that for laminar flow and can be neglected since it is usually less than 10%. With the correction factor the damping constant becomes

$$\eta = 0.133(v/a^2\Omega)^{1/4}(R/a)^{3/4}|\sigma-1|^{3/4} \times [1+0.089(a^2\Omega/v)^{1/4}(a/R)^{1/4}|\sigma-1|^{1/4}] \tag{39}$$

If the fluid is free from disturbances at entry the flow in a smooth pipe for some distance δ from the entry will be laminar even though turbulence develops further downstream. The Reynolds number at which the transition occurs may be expected to have the same order of magnitude as the Reynolds number for transition in flow along a flat plate. When the conditions are disturbed at entry the distance required for the velocity to take its final form is less but it depends on the amount of disturbance. When the flow is fully turbulent the inlet length δ has been found to be[8]

$$\delta = 1.386a\, Re^{1/4} \tag{40}$$

For $Re = 10^4$, $\delta = 13.86a$ which is considerably less than that for laminar flow. Thus the error which results from the assumption of steady flow in a pipe is less in the turbulent region than in the laminar region.

B) Flow Past a Flat Plate

1) *Laminar flow:* Since the flow for some distance from the entry is similar to boundary-layer flow past a flat plate a reasonable assumption is to treat the problem as boundary-layer flow as was done by Carrier and Miles.[1,2] The drag force on a flat plate of width b and length l is

$$D = 0.664bl^{1/2}\rho v^{1/2}U_\infty{}^{3/2} \tag{41}$$

From the dynamic analysis

$$D = c_d U_\infty = c_d R\dot{\beta} = \eta m\Omega R\dot{\beta} \tag{42}$$

The damping constant η becomes

$$\eta = 1.328(v/a^2\Omega)^{1/2}(|\sigma-1|^{1/2}/\gamma^{1/2}) \tag{43}$$

The question which now must be asked is: What is the distance or length required for the boundary layer to disappear? Defining the boundary-layer thickness ε as the distance for which $u = 0.99U_\infty$ then (Ref. 8, p. 122)

$$\varepsilon \approx 5(v)^{1/2}\delta/U_\infty \tag{44}$$

Setting ε equal to the radius of the pipe yields

$$\delta = (Re/25)a \tag{45}$$

which may or may not be a substantial portion of the fluid slug. The validity of each of the proposed methods is dependent on the length (fraction of fill) of the fluid, thus it may be that one is better for one system and the other better for another system.

2) *Turbulent flow:* The drag force on a flat plate of width b and length l when the boundary layer is turbulent is (see Ref. 7, p. 536).

$$D = 0.037\rho U_\infty{}^2 bl(U_\infty l/v)^{-1/5} \tag{46}$$

Equating this to the viscous drag force

$$D = c_d U_\infty = c_d R\dot{\beta}$$

one obtains

$$\eta = 0.074(v/a^2\Omega)^{1/5}(R/a)^{3/5}\gamma^{-1/5}|\sigma-1|^{4/5} \tag{47}$$

This equation is valid for

$$5\times10^5 < (R\gamma/2a)\, Re < 10^7 \tag{48}$$

Spin-Synchronous Mode

In this mode the velocity of the fluid is not constant but oscillatory with respect to the ring. The approach used by Leibold[6] to obtain a damping constant is to use the results of Bhuta and Koval,[7] who analyzed the damping of a satellite with a completely filled viscous ring damper mounted in a plane parallel to the spin axis. They modeled the motion of the fluid as a fluid in an infinite pipe with the pipe executing harmonic motion, and then obtained the energy dissipation rate which led to the damping constant.

To apply their analysis to this problem it is assumed that the analysis is valid for a finite length of fluid, the energy dissipation rate is then averaged over m cycles to determine the average rate of energy dissipation. This average rate is then used to calculate the damping constant. From Bhuta and Koval the energy dissipation/unit length at the end of the mth cycle is

$$E = \frac{2U^2\zeta\mu\pi}{s}\sum_{n=1}^{\infty}\frac{r_n^{\,6}}{(r_n^{\,4}+\zeta^2)^2}\left[\exp\left(\frac{-4r_n^{\,2}m\pi}{\zeta}\right)-1\right]+ \frac{U^2\mu m\pi^2}{s}\, i\{(i\zeta)^{1/2}I_0\bar{I}_1-(-i\zeta)^{1/2}\bar{I}_0 I_1\}/I_0\bar{I}_0 \tag{49}$$

where I_0 and I_1 are modified Bessel functions of the first kind and

$$I_j = I_j(i\zeta)^{1/2}$$
$$\bar{I}_j = I_j(-i\zeta)^{1/2} \tag{50}$$
$$\zeta = sa^2/v$$

U is the maximum velocity of the tube with respect to the fluid, a is the radius of the tube, s is the excitation frequency, and the r_n are the zeros of $J_o(r_n)$. Letting $g(\zeta)$ be the average amount of energy dissipation/cycle/unit mass/vel^2 one obtains for the damping constant

$$\eta = (|\lambda|/\pi)g(\zeta) \tag{51}$$

where

$$g(\zeta) = \frac{\pi}{\zeta} i\left\{\frac{(i\zeta)^{1/2}I_0\bar{I}_1-(-i\zeta)^{1/2}\bar{I}_0 I_1}{I_0\bar{I}_0}\right\}+\frac{2}{m}\sum_{i=1}^{\infty}\frac{r_n^{\,6}}{(r_n^{\,4}+\zeta^2)^2}\times \left[\exp\left(-\frac{4r_n^{\,2}m\pi}{\zeta}\right)-1\right] \tag{52}$$

$g(\zeta)$ is plotted for several values of m in Fig. 8. Since $g(\zeta)$ for $m = 20$ and $g(\zeta)$ for $m = \infty$ are approximately equal, it is reasonable to just use the value of $g(\zeta)$ when $m = \infty$ which is

$$g(\zeta) = \frac{\pi i}{\zeta}\left\{\frac{(i\zeta)^{1/2}I_0\bar{I}_1-(-i\zeta)^{1/2}\bar{I}_0 I_1}{I_0\bar{I}_0}\right\} \tag{53}$$

The excitation frequency s is

$$s = |\sigma - 1|\Omega$$

Recently some tests were run at NASA/GSFC for the Helios satellite.[10] An analysis of these tests results is given in Ref. 11. The analysis showed that all of the motion during the tests was in the spin synchronous mode, thus the methods suggested for calculating η for the nutation synchronous mode cannot be compared with experimental results. The test data analysis showed that η from the test results was consistently a factor of 4 to 5 higher than that predicted by Eq. (53). In the tests the ring was offset from the spin axis by 0.25 in. and gravity is acting on the slug. The gravitational effect can be removed[11] and numerical integration has shown that the offset has only a small effect on the time constant. Thus one has to conclude that the suggested method predicts a longer time constant than actually exists. More testing and more research in this area is needed.

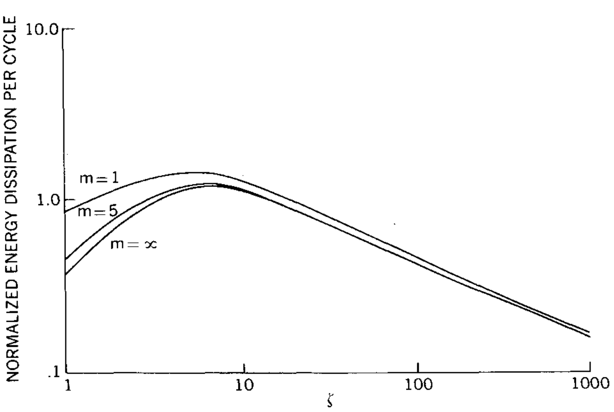

Fig. 8 Normalized energy dissipation per cycle as a function of the dimensionless parameter ζ.

As a means of comparing the suggested methods for calculating η in the nutation synchronous mode, η is calculated by the two methods for the parameters in one of the series of the tests. The fluid in the ring was mercury and the parameters for the first series of tests are $m = 0.152$ kg, $a = 0.28$ cm, $h = 30$ in., $R = 29$ cm, $v = 1.17\times10^{-3}$ cm^2/sec, $\bar{\gamma} = 0.25$, $A = 59.37$ slug ft^2, $\Omega = 95$ rpm, $\sigma = 0.337$, $\varepsilon = 6.35\times10^{-4}$, and $h = 2.63$. The Reynolds number is

$$Re = 2aR(1-\sigma)\Omega/v = 9.15\times10^4$$

thus the flow is in the turbulent region. The resulting values from Eqs. (38) and (46) are

Steady flow in a pipe

$$\eta = 0.135$$

Boundary-layer flow over a flat pipe

$$\eta = 0.053$$

In the spin-synchronous mode the value of η from the tests was $\eta = 0.21$ and the predicted value from Eq. (53) was $\eta = 0.044$.

4. Summary

The problem of a symmetric spinning satellite with a partially filled viscous ring damper has been investigated. It is shown that there are three possible modes of motion. In the mode for very small nutation angles, $\theta < 0.5°$, which was analyzed by Carrier and Miles, the fluid spreads out over the outer wall of the tube. In the other two modes, called the nutation-synchronous mode and spin-synchronous mode by Cartwright,[3,4] the fluid lumps together and behaves as a rigid slug. Approximate equations for time constants in the two modes have been developed and comparison with numerical integration of the exact equations shows that the approximations are good ones. Methods for calculating the damping constant in the two modes are also presented.

Appendix A: Equations of Motion

The system is assumed to consist of a symmetric rigid body (the satellite) to which is attached on the axis of symmetry a circular tube of radius R at a height h above the center of mass of the satellite. Moving in the tube is a rigid slug which fills a portion of the tube, the fraction fill is $\bar{\gamma}$. It is assumed that the center of mass of the system remains fixed at the satellite center of mass. Using the u, v, z coordinate system of Fig. 1 where the u axis passes through the center of mass of the fluid slug and the z axis is along the axis of symmetry the equations of motion are obtained by using the conservation of angular momentum of the system and Lagrange's equations for the motion of the rigid slug in the tube. The angular momentum of the system about the satellite center of mass is

$$\mathbf{H} = [(A+I_u)\omega_u - I_{uz}(\omega_z+\dot{\beta})]\mathbf{e}_u + (A+I_v)\omega_v\,\mathbf{e}_v + [C\omega_z + I_z(\omega_z+\dot{\beta}) - I_{uz}\,\omega_u]\mathbf{e}_z \tag{A1}$$

The kinetic energy of the system is

$$T = A(\omega_u^{\,2}+\omega_v^{\,2})/2 + C\omega_z^{\,2}/2 + [I_u\omega_u^{\,2} + I_v\omega_v^{\,2} + I_z(\omega_z+\dot{\beta})^2 - 2I_{uz}\,\omega_u(\omega_z+\dot{\beta})]/2 \tag{A2}$$

where

$$I_u = m[h^2 + (1-\sin\gamma/\gamma)R^2/2]$$
$$I_v = m[h^2 + (1+\sin\gamma/\gamma)R^2/2]$$
$$I_z = mR^2$$
$$I_{uz} = mhR\sin(\gamma/2)/(\gamma/2)$$

Assuming that the only force acting is a linear viscous damping force which results from the motion of the fluid slug relative to the satellite, the generalized force becomes

$$Q_\beta = -c_d R^2\dot{\beta} \tag{A4}$$

It is advantageous in this problem to introduce dimensionless variables and to express the parameters of the system in terms of suitable dimensionless parameters. Let the dimensionless time τ be

$$\tau = \Omega t \quad \text{(A5)}$$

where Ω is the initial spin rate and let the dimensionless angular velocity components be p, q, r where

$$\omega_u = p\Omega, \qquad \omega_v = q\Omega, \qquad \omega_z = r\Omega \quad \text{(A6)}$$

The dimensionless parameters are: σ the ratio of the spin axis moment of inertia to the transverse moment of inertia, the fraction fill $\bar{\gamma}$ or the angle of fill γ, b the ratio of the height of the ring above the satellite center of mass to the radius of the ring, a dimensionless damping constant η, and ε which is the ratio of the inertia of the tube filled with fluid to the transverse moment of inertia of the satellite.

$$\sigma = C/A, \qquad b = h/R, \qquad \eta = c_d/m\Omega, \qquad \varepsilon = mR^2/A\bar{\gamma} \quad \text{(A7)}$$

Application of the conservation of angular momentum equation $\dot{\mathbf{H}} = 0$ and Lagrange's equations for quasi-coordinates

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{\beta}} - \omega_v \frac{\partial T}{\partial \omega_u} + \omega_u \frac{\partial T}{\partial \omega_v} = Q_\beta \quad \text{(A8)}$$

yields

$$p' + (\lambda r - \beta')q/D_1 + [A_1(r+\beta') - A_2 p]q/D_1 + A_3\beta'/\Omega D_1 = 0 \quad \text{(A9a)}$$

$$q' - (\lambda r - \beta')p/D_2 - [B_1(r+\beta') + B_2 p]p/D_2 + B_2(r+\beta')^2/D_2 = 0 \quad \text{(A9b)}$$

$$\beta'' + C_1\beta'/\Omega + C_2 pq + C_3[q(r+\beta') - p'] = 0 \quad \text{(A9c)}$$

$$r = 1 + C_4\beta/\Omega \quad \text{(A9d)}$$

where primes denote differentiation with respect to τ and

$$\lambda = \sigma - 1$$

$$A_1 = \varepsilon\bar{\gamma}\left[-b^2 + \left(1 - \frac{\sin\gamma}{\gamma}\right)\Big/2\right] + \frac{\varepsilon b^2 \sin(\gamma/2)}{\pi}\frac{\sin(\gamma/2)}{(\gamma/2)}$$

$$A_2 = [\sin\gamma/\gamma) - 1]\,\varepsilon b \sin(\gamma/2)/\pi$$

$$A_3/\Omega = \varepsilon b \eta \bar{\gamma}[\sin(\gamma/2)]/(\gamma/2) \quad \text{(A10)}$$

$$B_1 = \varepsilon\bar{\gamma}[-b^2 + (1 + \sin\gamma/\gamma)/2]$$

$$B_2 = -\varepsilon b \sin(\gamma/2)/\pi$$

$$C_1/\Omega = (1 + \varepsilon\bar{\gamma}/\sigma)\eta$$

$$C_2 = \sin\gamma/\gamma$$

$$C_3 = b[\sin(\gamma/2)]/(\gamma/2)$$

$$C_4/\Omega = \varepsilon\eta\bar{\gamma}/\sigma$$

$$D_1 = 1 + \varepsilon\bar{\gamma}\left[b^2 + \left(1 - \frac{\sin\gamma}{\gamma}\right)\Big/2\right] - \frac{\varepsilon b^2 \sin(\gamma/2)}{\pi}[\sin(\gamma/2)]/(\gamma/2)$$

$$D_2 = 1 + \varepsilon\bar{\gamma}[b^2 + (1 + \sin\gamma/\gamma)/2]$$

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